

矢量介子自旋的一般约束

Xin-Li Sheng

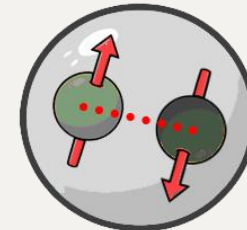
盛欣力



第二届旋转与磁场下的QCD物质研讨会
2026年9月11日——14日



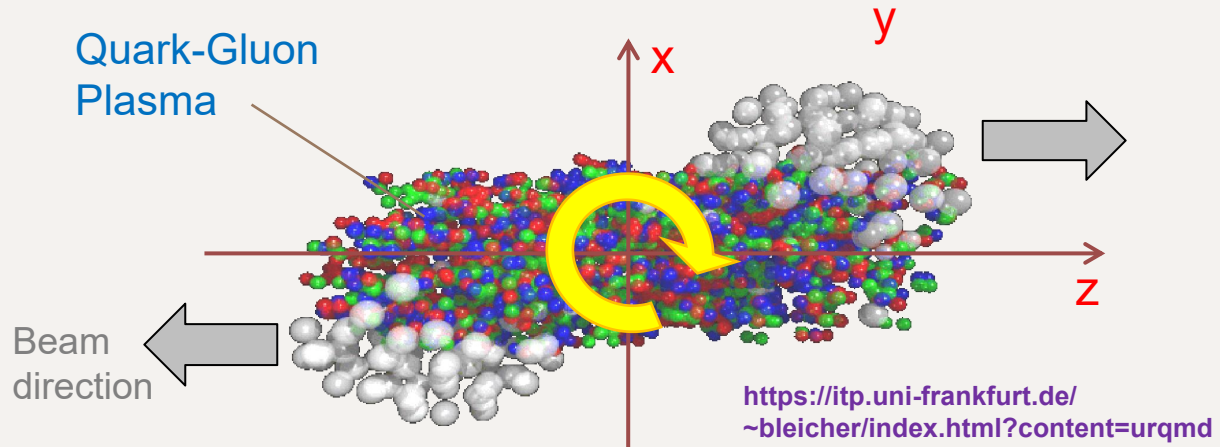
- Spin in heavy-ion collisions
- Spin alignment induced by anisotropy
- General structure of tensor polarization
- Summary



Heavy-ion collisions



Strongly interacting matter with vorticity and magnetic fields



Initial orbital angular momentum

Vorticity field
Magnetic field
Strong field

Polarized quark/gluon



Spin polarization for baryons,
 Λ , Σ^0 , Δ^{++} , Ω^- , ...

Spin alignment for vector mesons,
 ϕ , K^{*0} , ρ^0 , J/ψ , ...

S. A. Voloshi, nucl-th/0410089

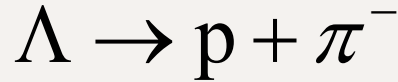
Z.-T. Liang, X.-N. Wang, PRL 94, 102301 (2005) [Erratum: PRL 96, 039901 (2006)]; PLB 629, 20 (2005)

F. Becattini, F. Piccinini, J. Rizzo, PRC 76, 044901 (2007)

Λ spin polarization



- Spin polarization of Λ hyperons



(uds) (uud) ($\bar{u}d$)

strangeness 1 0 0

Weak decay

Parity violation

- Angle distribution of daughter protons in Λ 's rest frame

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} \left(1 + \alpha_H \mathcal{P}_\Lambda \frac{\mathbf{n}^* \cdot \mathbf{p}^*}{|\mathbf{p}^*|} \right)$$

decay parameter
(constant)

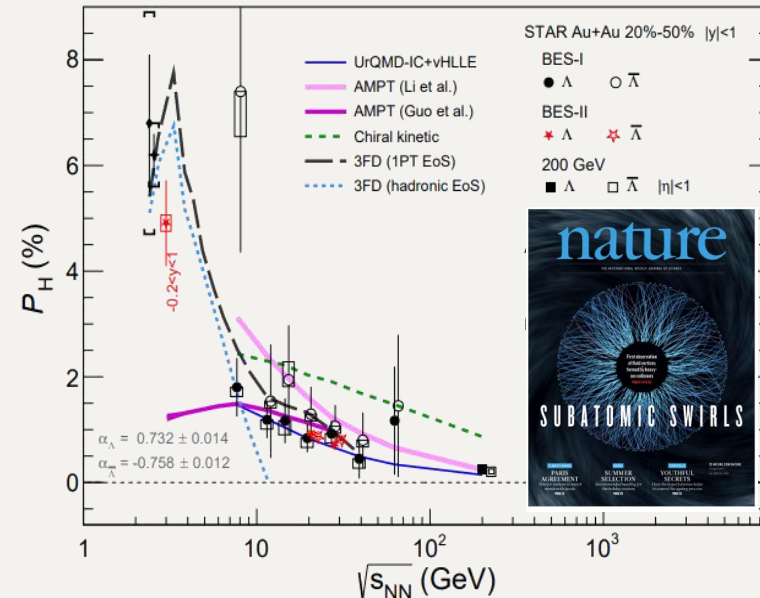
Λ 's spin
polarization

$\cos\theta^*$

- Spin density matrix

$$\rho = \begin{pmatrix} \rho_{\frac{1}{2}\frac{1}{2}} & \rho_{\frac{1}{2},-\frac{1}{2}} \\ \rho_{-\frac{1}{2}\frac{1}{2}} & \rho_{-\frac{1}{2},-\frac{1}{2}} \end{pmatrix} = 1 + \boldsymbol{\sigma} \cdot \mathbf{P}$$

$$P_3 = \rho_{\frac{1}{2}\frac{1}{2}} - \rho_{-\frac{1}{2},-\frac{1}{2}}$$



Becattini, Buzzegoli, Niida, Pu, Tang, IJMPA 2024
STAR, Nature 648 (2017) 62

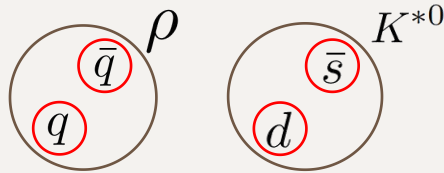
➡ Related talks: Zhen-Yu Chen,
Jin-Long Zhang, Xing-Rui Gou

Vector meson spin alignment



- Spin density matrix for a vector meson ($J^P = 1^-$)

$$\rho_{rs}^{S=1} = \begin{pmatrix} \rho_{+1,+1} & \rho_{+1,0} & \rho_{+1,-1} \\ \rho_{0,+1} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,+1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$



Spin alignment
= 1/3 for unpolarized meson

- A meson's spin is determined by its constituent q & $\bar{q}$

Z.-T. Liang & X.-N. Wang, PLB 629, 20 (2005)

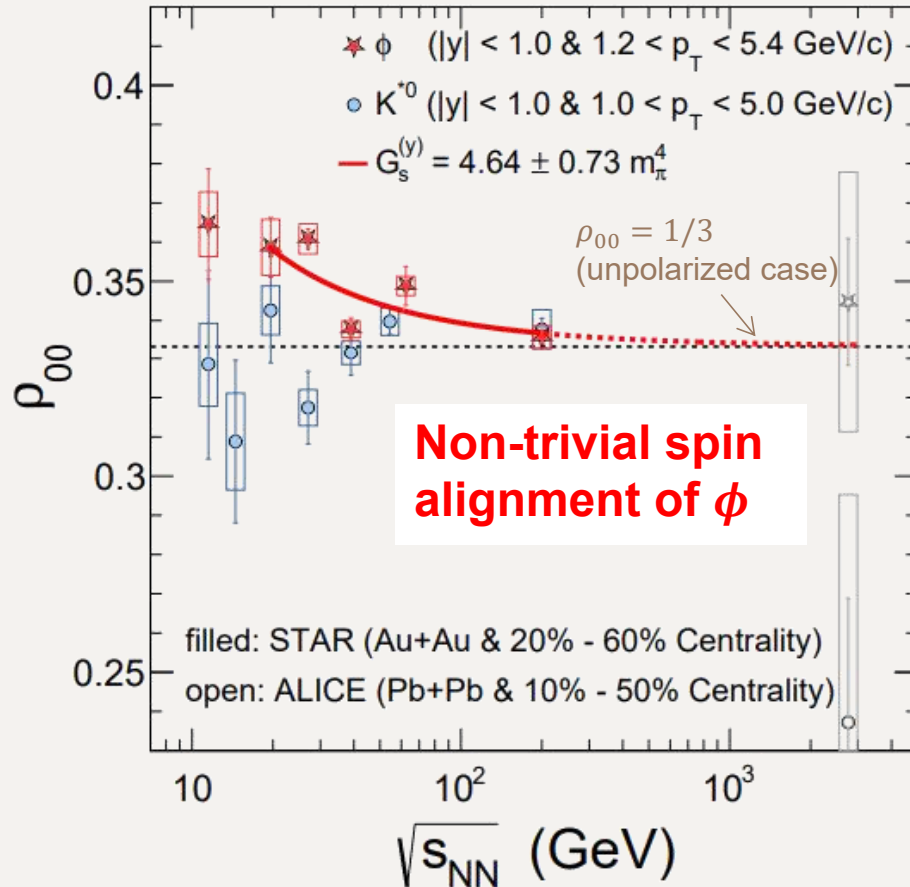
$$\rho_{00}^{\rho(\text{rec})} = \frac{1 - P_q^2}{3 + P_q^2}$$

$$\rho_{00}^{K^*(\text{rec})} = \frac{1 - P_q P_s}{3 + P_q P_s}$$

Processes	Examples	Polar angle distribution $W(\theta)$	Spin is converted to
Strong p -wave decay	$K^{*0} \rightarrow K^+ + \pi^-$ $\phi \rightarrow K^+ + K^-$	$\frac{3}{4} [1 - \rho_{00} + (3\rho_{00} - 1) \cos^2 \theta]$	OAM
Dilepton decay	$J/\psi \rightarrow \mu^+ + \mu^-$	$\frac{3}{8} [1 + \rho_{00} + (1 - 3\rho_{00}) \cos^2 \theta]$	Spin

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].
P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)

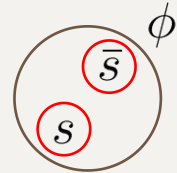
Global spin alignment



STAR, Nature 614, 244 (2023)

➡ Related talk: Sheng-Qin Feng,
Xiao-Zhi Bai, Rui-Xiang Chen

- Non-relativistic spin coalescence



- Spin alignment

↔ Anisotropy of spin correlation

$$\rho_{00} = \frac{1}{3} + \frac{4}{9} \left[\frac{1}{2} \langle \mathbf{P}_q^T \cdot \mathbf{P}_{\bar{q}}^T \rangle - \langle P_q^L P_{\bar{q}}^L \rangle \right]$$

X.-L. Xia, X.-G. Huang, PLB 817, 136325 (2021)

J.-H. Chen, Z.-T. Liang, Y.-G. Ma, XLS, Q. Wang,
Sci.China Phys.Mech.Astron. 68, 211001 (2025).

- Internal spin-spin interaction

➡ { Significant $q\bar{q}$ spin correlation
No spin alignment

$$\Delta H = -\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} = -\frac{1}{2} [(\mathbf{S}_q + \mathbf{S}_{\bar{q}})^2 - \mathbf{S}_q^2 - \mathbf{S}_{\bar{q}}^2]$$

Independent of S_y

ϕ meson spin alignment



- Spin alignment of ϕ meson measured along direction of ϵ_0

$$\rho_{00} \approx \frac{1}{3} + C_1 \left[\frac{1}{3} \omega' \cdot \omega' - (\epsilon_0 \cdot \omega')^2 \right] + C_1 \left[\frac{1}{3} \epsilon' \cdot \epsilon' - (\epsilon_0 \cdot \epsilon')^2 \right] \quad \text{small}$$

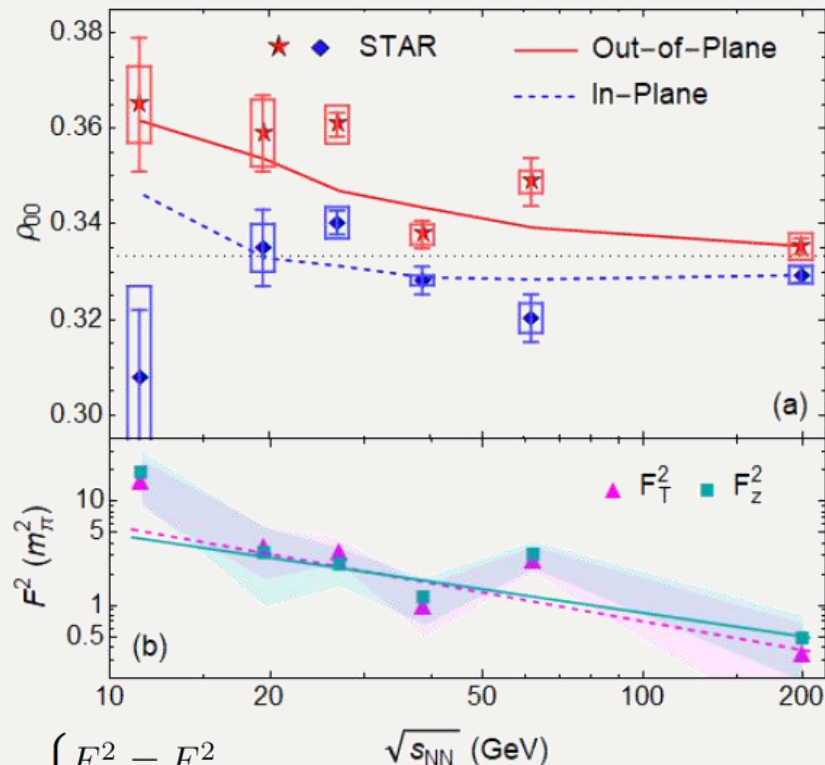
$$- \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_1 \left[\frac{1}{3} \mathbf{B}'_\phi \cdot \mathbf{B}'_\phi - (\epsilon_0 \cdot \mathbf{B}'_\phi)^2 \right]$$

$$- \frac{4g_\phi^2}{m_\phi^2 T_h^2} C_2 \left[\frac{1}{3} \mathbf{E}'_\phi \cdot \mathbf{E}'_\phi - (\epsilon_0 \cdot \mathbf{E}'_\phi)^2 \right]$$

Lorentz transform

Fields in lab frame

$$\langle g_\phi^2 \mathbf{B}_\phi^i \mathbf{B}_\phi^j / T_h^2 \rangle = \langle g_\phi^2 \mathbf{E}_\phi^i \mathbf{E}_\phi^j / T_h^2 \rangle = F^2 \delta^{ij} + \Delta \hat{\mathbf{a}}^i \hat{\mathbf{a}}^j \quad \begin{cases} F_T^2 = F^2 \\ F_z^2 = F^2 + \Delta \end{cases}$$



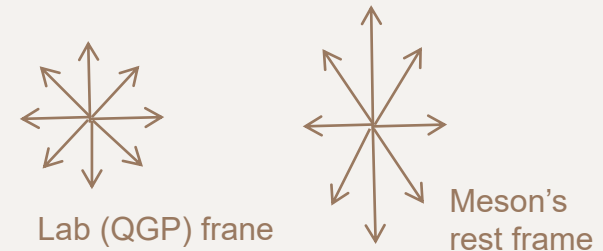
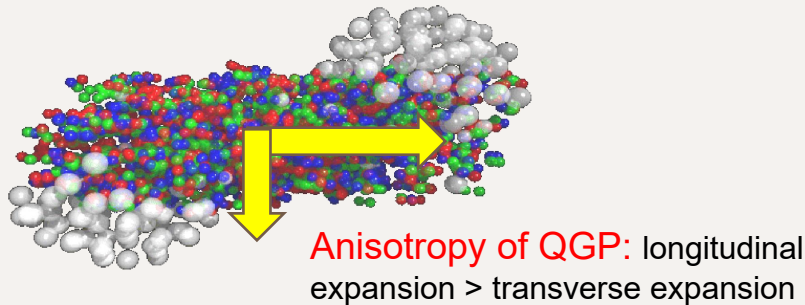
- Cancellation for mixing terms (because of CP and reflection symmetries)
- All field contributions appear quadratically, spin alignment measures **anisotropy of fluctuations in meson's rest frame**

XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024).

Anisotropy of fluctuations

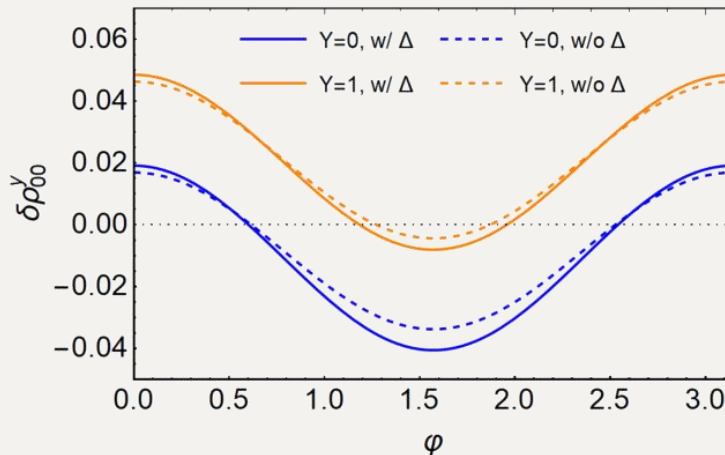


- Spin alignment measures **anisotropy of fluctuations** in meson's rest frame



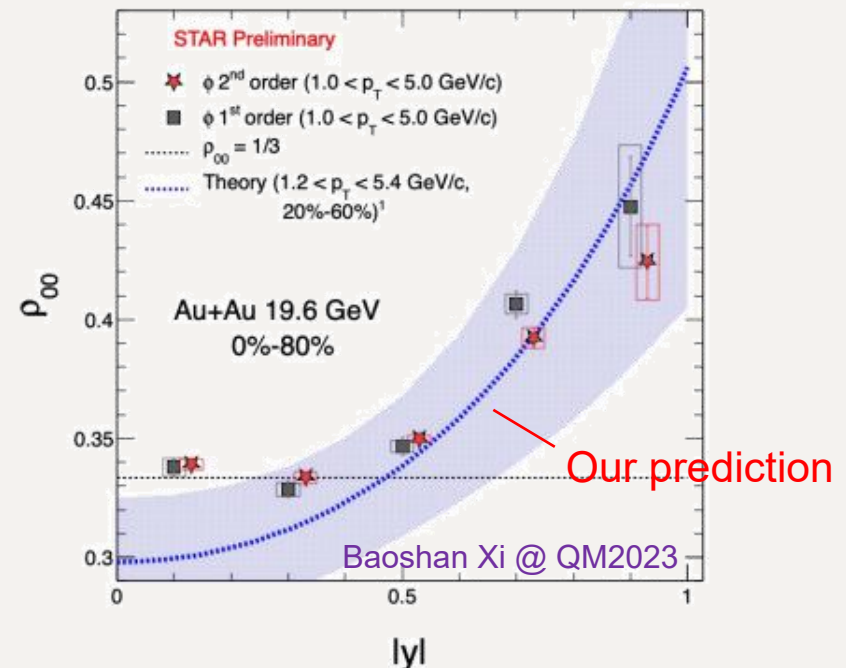
Anisotropy induced by motion relative to QGP background

- Predictions for differential measurements



The extracted $\Delta \ll F^2$ and its effect on azimuthal-angle dependence is relatively small

XLS, S. Pu, Q. Wang, PRC 108, 054902 (2023)

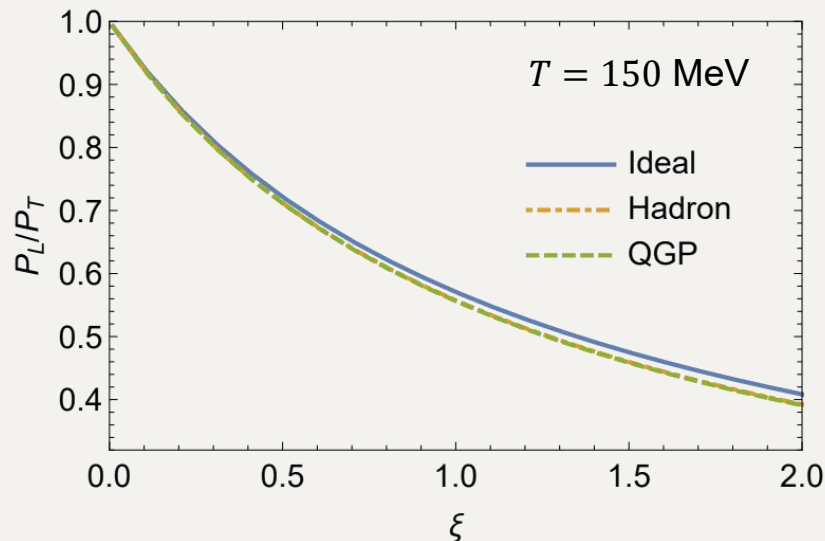


- Anisotropic particle distribution (Romatschke-Strickland distribution)

$$f_i^{\text{RS}}(p, \xi) = \frac{d_s}{\exp \left[\beta \sqrt{p_T^2 + (\xi + 1)p_z^2 + m_i^2} \right] \pm 1}$$

- Energy-momentum tensor for an ideal hydro with an anisotropy pressure

$$\begin{aligned} T^{\mu\nu}(\xi) &= \sum_i \int \frac{d^3\mathbf{p}}{(2\pi)^3 \sqrt{\mathbf{p}^2 + m_i^2}} p^\mu p^\nu f_i^{\text{RS}}(p, \xi) \\ &= (\varepsilon + P_T) u^\mu u^\nu - g^{\mu\nu} P_T - (P_T - P_L) l^\mu l^\nu \end{aligned}$$



Ideal: massless particles

Hadron: hadron gas (π , K , p , n)

QGP: u , d , s , g

relationship between ξ and P_L/P_T
is nearly model-independent.

Spin alignment



- Cross section for spin- λ vector meson

$$\sigma_\lambda(p, q) = \epsilon_\mu^*(p, \lambda) \epsilon_\nu(p, \lambda) \mathcal{M}^{\mu\nu}(p, q)$$

$$\mathcal{M}^{\mu\nu}(p, q) = c_1 m_V^2 g^{\mu\nu} + c_2 q^\mu q^\nu + c_3 p^\mu p^\nu + c_4 (p^\mu q^\nu + q^\mu p^\nu) + \underbrace{ic_5(p^\mu q^\nu - q^\mu p^\nu)}_{\text{Perpendicular to polarization vector}} + \underbrace{ic_6 \epsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}_{\text{Antisymmetric}}$$

- Spin alignment in y-direction ($|\mathbf{p}|, |\mathbf{q}| \ll m_V$)

$$\delta\rho_{00}^y(p) \approx \frac{\langle q_y^2 \rangle - \langle \mathbf{q}^2 \rangle / 3}{\langle \mathbf{q}^2 \rangle - 3 m_V^2 c_1 / c_2}$$

$$\langle q_y^2 \rangle \neq \langle \mathbf{q}^2 \rangle / 3$$

Anisotropic distributions

$$\langle A \rangle \equiv \frac{\int d^3\mathbf{q} / (E_{p_1} E_{p_2}) \delta(E_p - E_{p_1} - E_{p_2}) A f_1(p_1) f_2(p_2)}{\int d^3\mathbf{q} / (E_{p_1} E_{p_2}) \delta(E_p - E_{p_1} - E_{p_2}) f_1(p_1) f_2(p_2)}$$

- Coefficients in transition amplitude rely on **production mechanisms** for vector meson

$$K^+ + K^- \rightarrow \phi$$

$$\mathcal{L}_{\phi KK} = g_{\phi KK} \phi^\mu (K^+ \partial_\mu K^- - K^- \partial_\mu K^+)$$

$$s + \bar{s} \rightarrow \phi$$

$$\begin{aligned} \mathcal{L}_\phi &= ig_{\phi s \bar{s}} \bar{\psi}_s \left[\gamma^\mu + \frac{\zeta_\phi}{2M} \sigma^{\mu\nu} (\vec{\partial}_\nu + \overleftarrow{\partial}_\nu) \right] \psi_s \phi_\mu \\ &= g_{\phi s \bar{s}} \bar{\psi}_s \left[i\gamma^\mu \left(1 + \zeta_\phi \frac{m_s}{M} \right) + \frac{\zeta_\phi}{2M} \overleftrightarrow{\partial}^\mu \right] \psi_s \phi_\mu \end{aligned}$$

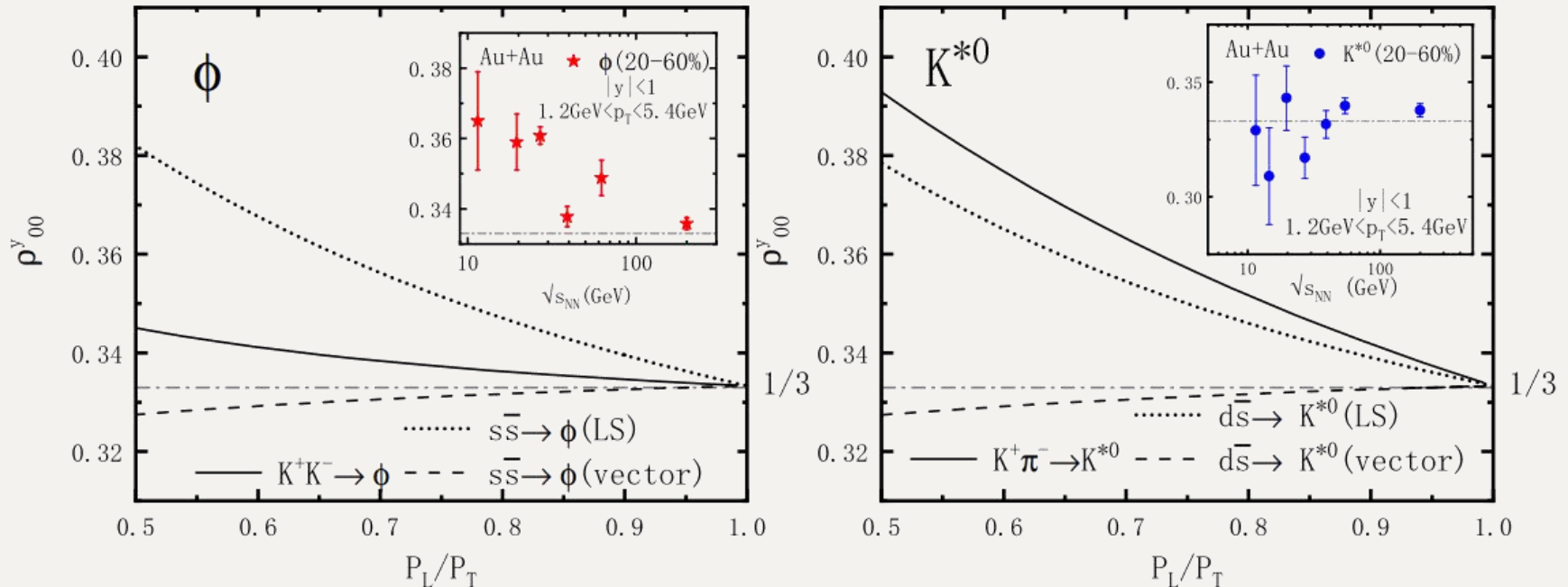
Vector

Spin-orbit (LS)

Spin alignment



- Spin alignment induced by anisotropic distributions as a function of P_L/P_T



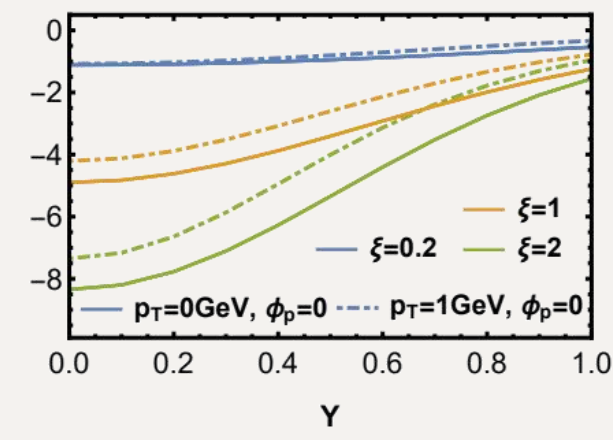
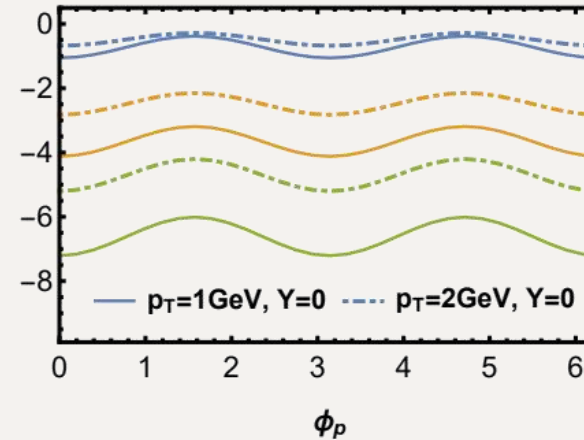
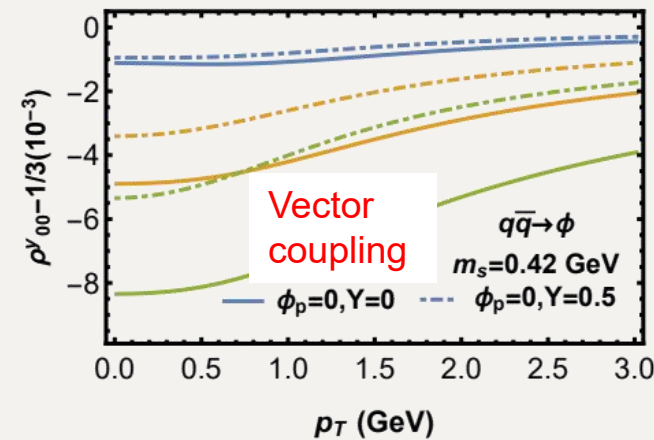
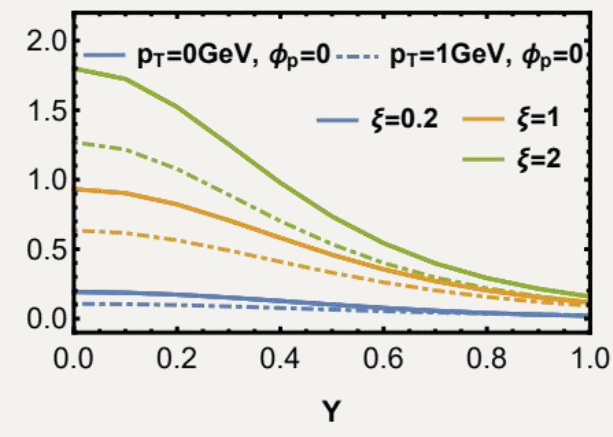
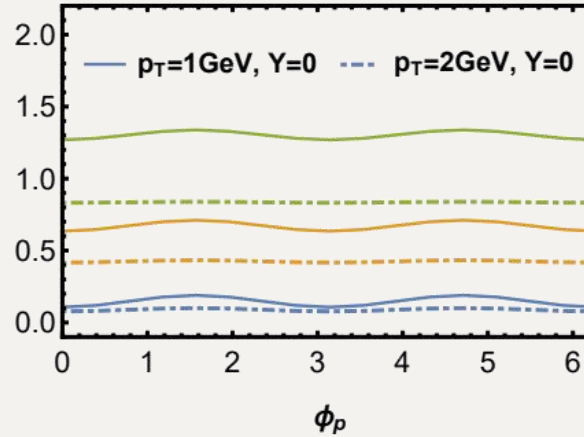
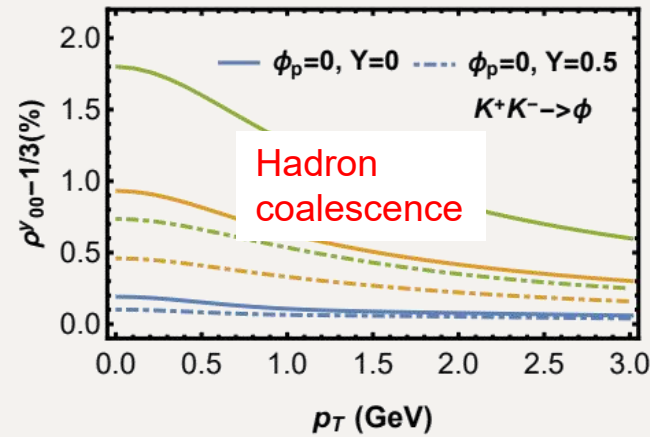
➤ Not from $q\bar{q}$ spin correlation

➤ Model-dependent

➤ Can be $O(10^{-2})$ if fluid background is highly-anisotropic

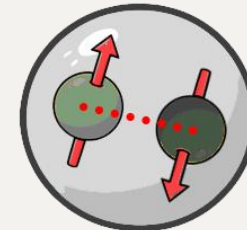
W.-B. Dong, XLS, Y.-L. Yin, D. H. Rischke,
Q. Wang, arXiv: 2607.06138

Momentum dependence



$s + \bar{s} \rightarrow \phi$ with pure LS coupling is similar to $K^+ + K^- \rightarrow \phi$

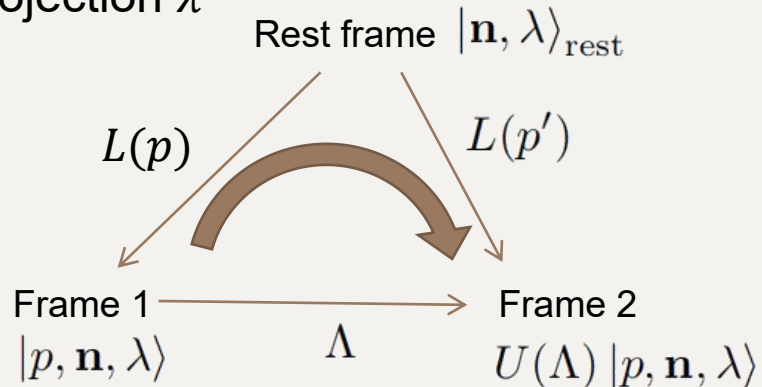
- Spin in heavy-ion collisions
- Spin alignment induced by anisotropy
- **General structure of tensor polarization**
- Summary



- For a particle with momentum p^μ and spin projection λ

$$U(\Lambda) |p, \mathbf{n}, \lambda\rangle = |p', \mathbf{n}, \lambda'\rangle D_{\lambda'\lambda}[R(\Lambda, p)]$$

Wigner rotation $R(\Lambda, p) \equiv L^{-1}(p') \Lambda L(p)$



- Wigner little group $[SO(3)]$** leaves rest-frame momentum $p_{\text{rest}}^\mu = (m, 0, 0, 0)$ invariant

- Spin density matrix

$$\rho^{\mathbf{n}}(p) \rightarrow \rho^{\mathbf{n}'}(p') = D[R(\Lambda, p)] \rho^{\mathbf{n}}(p) D^\dagger[R(\Lambda, p)] \quad \text{Similarity transformation of } SO(3)$$

Irreducible decomposition

$$1 \otimes 1 = 0 \oplus 1 \oplus 2$$

$$\rho_{\lambda\lambda'}^{\mathbf{n}}(p) = \frac{1}{3} I_3 + \frac{1}{2} \mathbf{P}^{\mathbf{n}} \cdot \mathbf{S} + \frac{1}{4} \mathcal{T}_{ij}^{\mathbf{n}} \Sigma_{ij}$$

$$P_i^{\mathbf{n}} \rightarrow P_i^{\mathbf{n}'} = R_{ij} P_j^{\mathbf{n}} \quad \text{Vector polarization}$$

$$\mathcal{T}_{ij}^{\mathbf{n}} \rightarrow \mathcal{T}_{ij}^{\mathbf{n}'} = R_{ik} R_{jl} \mathcal{T}_{kl}^{\mathbf{n}} \quad \text{Tensor polarization}$$

$$S_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Sigma_{ij} \equiv (S_i S_j + S_j S_i) - \frac{2}{3} \mathbf{S}^2 \delta_{ij}$$

Spin-1 representation of $SO(3)$ generators in angular momentum basis

How to measure?



- Vector polarization

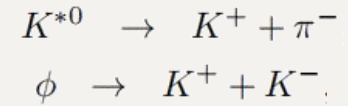
$$\begin{aligned} P_1^{\mathbf{n}} &= \sqrt{2}\text{Re}(\rho_{10}^{\mathbf{n}} + \rho_{0,-1}^{\mathbf{n}}) \\ P_2^{\mathbf{n}} &= -\sqrt{2}\text{Im}(\rho_{10}^{\mathbf{n}} + \rho_{0,-1}^{\mathbf{n}}) \\ P_3^{\mathbf{n}} &= \rho_{11}^{\mathbf{n}} - \rho_{-1,-1}^{\mathbf{n}} \end{aligned}$$

- Tensor polarization

$$\begin{aligned} \mathcal{T}_{12}^{\mathbf{n}} &= -2\text{Im}\rho_{1,-1}^{\mathbf{n}} \\ \mathcal{T}_{23}^{\mathbf{n}} &= -\sqrt{2}\text{Im}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \\ \mathcal{T}_{31}^{\mathbf{n}} &= \sqrt{2}\text{Re}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \\ \mathcal{T}_{11}^{\mathbf{n}} &= \rho_{00}^{\mathbf{n}} - \frac{1}{3} + 2\text{Re}\rho_{1,-1}^{\mathbf{n}} \\ \mathcal{T}_{22}^{\mathbf{n}} &= \rho_{00}^{\mathbf{n}} - \frac{1}{3} - 2\text{Re}\rho_{1,-1}^{\mathbf{n}} \\ \mathcal{T}_{33}^{\mathbf{n}} &= -2(\rho_{00}^{\mathbf{n}} - \frac{1}{3}) \end{aligned}$$

$$\mathcal{T}_{11}^{\mathbf{n}} + \mathcal{T}_{22}^{\mathbf{n}} + \mathcal{T}_{33}^{\mathbf{n}} = 0$$

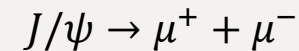
- Strong p -wave decay



$$\begin{aligned} \frac{dN}{d\Omega} = \frac{3}{8\pi} &\left[(1 - \rho_{00}^{\mathbf{n}}) + (3\rho_{00}^{\mathbf{n}} - 1) \cos^2 \theta \right. \\ &- 2\text{Re}\rho_{1,-1}^{\mathbf{n}} \sin^2 \theta \cos 2\phi + 2\text{Im}\rho_{1,-1}^{\mathbf{n}} \sin^2 \theta \sin 2\phi \\ &- \sqrt{2}\text{Re}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \sin 2\theta \cos \phi \\ &\left. + \sqrt{2}\text{Im}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \sin 2\theta \sin \phi \right] \end{aligned}$$

Direction of $\mathbf{n}$ matters!

- Dilepton decay



$$\begin{aligned} \frac{dN}{d\Omega} = \frac{3}{16\pi} &\left[(1 + \rho_{00}^{\mathbf{n}}) + (1 - 3\rho_{00}^{\mathbf{n}}) \cos^2 \theta \right. \\ &+ 2\text{Re}\rho_{1,-1}^{\mathbf{n}} \sin^2 \theta \cos 2\phi - 2\text{Im}\rho_{1,-1}^{\mathbf{n}} \sin^2 \theta \sin 2\phi \\ &+ \sqrt{2}\text{Re}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \sin 2\theta \cos \phi \\ &\left. - \sqrt{2}\text{Im}(\rho_{10}^{\mathbf{n}} - \rho_{0,-1}^{\mathbf{n}}) \sin 2\theta \sin \phi \right] \end{aligned}$$

K. Schilling, P. Seyboth, G. E. Wolf, NPB 15, 397 (1970) [Erratum-ibid. B 18, 332 (1970)].
P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)

Off-diagonal elements



- Off-diagonal elements of vector meson density matrix (spin quantization direction is set to z-direction)

$$\rho_{00}^{\hat{z}}(\mathbf{x}, \mathbf{p}) \approx \frac{1}{3} - \frac{2}{3} \left\langle P_q^z P_{\bar{q}}^z - \frac{1}{3} \mathbf{P}_q \cdot \mathbf{P}_{\bar{q}} \right\rangle_V$$

Difference between z-z correlation and x-x (y-y) correlation

$$\text{Re} \rho_{1,-1}^{\hat{z}}(\mathbf{x}, \mathbf{p}) \approx \frac{1}{3} \langle P_q^x P_{\bar{q}}^x - P_q^y P_{\bar{q}}^y \rangle_V$$

Difference between x-x correlation and y-y correlation

$$-\text{Im} \rho_{1,-1}^{\hat{z}}(\mathbf{x}, \mathbf{p}) \approx \frac{1}{3} \langle P_q^x P_{\bar{q}}^y + P_q^y P_{\bar{q}}^x \rangle_V$$

x-y correlation

$$\text{Re} (\rho_{10}^{\hat{z}} - \rho_{0,-1}^{\hat{z}})(\mathbf{x}, \mathbf{p}) \approx \frac{\sqrt{2}}{3} \langle P_q^x P_{\bar{q}}^z + P_q^z P_{\bar{q}}^x \rangle_V$$

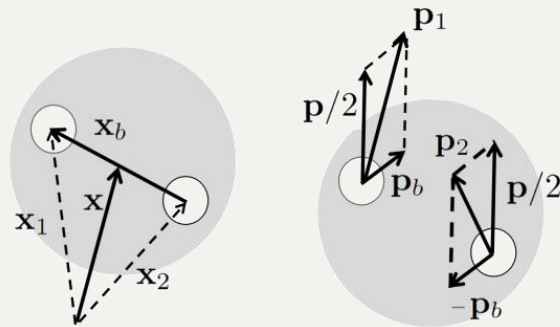
x-z correlation

$$-\text{Im} (\rho_{10}^{\hat{z}} - \rho_{0,-1}^{\hat{z}})(\mathbf{x}, \mathbf{p}) \approx \frac{\sqrt{2}}{3} \langle P_q^y P_{\bar{q}}^z + P_q^z P_{\bar{q}}^y \rangle_V$$

y-z correlation

- Average over constituent $q/\bar{q}$'s relative position and relative momentum

$$\begin{aligned} \langle P_q^i P_{\bar{q}}^j \rangle_V &\equiv \frac{1}{\pi^3} \int d^3 \mathbf{x}_b d^3 \mathbf{p}_b \exp \left(-\frac{\mathbf{p}_b^2}{a_V^2} - a_V^2 \mathbf{x}_b^2 \right) \\ &\times P_q^i(\mathbf{x}_1, \mathbf{p}_1) P_{\bar{q}}^j(\mathbf{x}_2, \mathbf{p}_2) \end{aligned}$$



X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)

XLS, Q. Wang, X.-N. Wang, Phys.Rev.D 102 (2020) 5, 056013

J.-H. Chen, Z.-T. Liang, Y.-G. Ma, XLS, Q. Wang, arXiv:2407.06480

Change of quantization axis



- State with spin projection λ in direction $\mathbf{n}$

$$\mathbf{n} \cdot \hat{\mathbf{J}} |\mathbf{n}, \lambda\rangle_{\text{rest}} = \lambda |\mathbf{n}, \lambda\rangle_{\text{rest}}$$

- Consider a spatial rotation R which transforms $\mathbf{n}$ to $\mathbf{n}' = R\mathbf{n}$

$$\begin{aligned} \mathbf{n}' \cdot \hat{\mathbf{J}} U(R) |\mathbf{n}, \lambda\rangle_{\text{rest}} &= U(R) \left[U^\dagger(R) \mathbf{n}' \cdot \hat{\mathbf{J}} U(R) \right] |\mathbf{n}, \lambda\rangle_{\text{rest}} \\ &= U(R) \left[(R\mathbf{n}) \cdot (R\hat{\mathbf{J}}) \right] |\mathbf{n}, \lambda\rangle_{\text{rest}} \\ &= U(R) (\mathbf{n} \cdot \hat{\mathbf{J}}) |\mathbf{n}, \lambda\rangle_{\text{rest}} \\ &= \lambda U(R) |\mathbf{n}, \lambda\rangle_{\text{rest}} \end{aligned}$$

$$|\mathbf{n}', \lambda'\rangle_{\text{rest}} = U(R) |\mathbf{n}, \lambda'\rangle_{\text{rest}} = |\mathbf{n}, \lambda\rangle_{\text{rest}} D_{\lambda\lambda'}(R)$$

$$D_{\lambda\lambda'}(R) = {}_{\text{rest}} \langle \mathbf{n}, \lambda | \mathbf{n}', \lambda' \rangle_{\text{rest}} = \epsilon_\lambda^* \cdot \epsilon'_{\lambda'}$$

$$\epsilon_0 = \mathbf{n}, \quad \epsilon_{\pm 1} = -\frac{1}{\sqrt{2}}(\pm \mathbf{n}_1 + i\mathbf{n}_2), \quad \mathbf{n}_1 \perp \mathbf{n}_2 \perp \mathbf{n}$$

- Total density matrix is independent to choice of quantization axis

Rotation of spin density matrix

$$\rho^{\mathbf{n}'} = D^\dagger(R) \rho^{\mathbf{n}} D(R)$$

Change of quantization axis

- Spin polarization vectors when quantization directions are in x -, y -, or z -directions

$$\epsilon_0 = (1, 0, 0), \quad \epsilon_{+1} = -\frac{1}{\sqrt{2}}(0, 1, i), \quad \epsilon_{-1} = \frac{1}{\sqrt{2}}(0, 1, -i),$$

$$\epsilon_0 = (0, 1, 0), \quad \epsilon_{+1} = -\frac{1}{\sqrt{2}}(i, 0, 1), \quad \epsilon_{-1} = \frac{1}{\sqrt{2}}(-i, 0, 1)$$

$$\epsilon_0 = (0, 0, 1), \quad \epsilon_{+1} = -\frac{1}{\sqrt{2}}(1, i, 0), \quad \epsilon_{-1} = \frac{1}{\sqrt{2}}(1, -i, 0),$$

- Rotating matrices

$$D(R_{\hat{y} \rightarrow \hat{x}}) = \begin{pmatrix} \frac{i}{2} & \frac{i}{\sqrt{2}} & \frac{i}{2} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{2} & \frac{i}{\sqrt{2}} & -\frac{i}{2} \end{pmatrix}$$

$$D(R_{\hat{y} \rightarrow \hat{z}}) = \begin{pmatrix} -\frac{i}{2} & -\frac{1}{\sqrt{2}} & \frac{i}{2} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ -\frac{i}{2} & \frac{1}{\sqrt{2}} & \frac{i}{2} \end{pmatrix}$$

$$\left. \begin{aligned} \rho_{00}^{\hat{z}} &= \frac{1}{2} \left(1 - \rho_{00}^{\hat{y}} - 2\text{Re}\rho_{1,-1}^{\hat{y}} \right) \\ \rho_{00}^{\hat{x}} &= \frac{1}{2} \left(1 - \rho_{00}^{\hat{y}} + 2\text{Re}\rho_{1,-1}^{\hat{y}} \right) \end{aligned} \right\}$$

$$\rho_{00}^{\hat{x}} + \rho_{00}^{\hat{y}} + \rho_{00}^{\hat{z}} = 1,$$

$$\text{Re}\rho_{1,-1}^{\hat{y}} = \frac{1}{2}\rho_{00}^{\hat{y}} + \rho_{00}^{\hat{x}} - \frac{1}{2}$$

Measuring off-diagonal elements



- Can we express off-diagonal elements as linear combinations of ρ_{00} in different directions?

YES

$$\text{Re}\rho_{1,-1}^{\hat{y}} = \frac{1}{2}\rho_{00}^{\hat{y}} + \rho_{00}^{\hat{x}} - \frac{1}{2}$$

$$-2\text{Im}\rho_{1,-1}^{\hat{y}} = 1 - 2\rho_{00}^{(\hat{x}+\hat{z})/\sqrt{2}} - \rho_{00}^{\hat{y}}$$

$$-\sqrt{2}\text{Im}(\rho_{10}^{\hat{y}} - \rho_{0,-1}^{\hat{y}}) = 1 - 2\rho_{00}^{(\hat{x}+\hat{y})/\sqrt{2}} - \rho_{00}^{\hat{z}}$$

$$\sqrt{2}\text{Re}(\rho_{10}^{\hat{y}} - \rho_{0,-1}^{\hat{y}}) = 1 - 2\rho_{00}^{(\hat{y}+\hat{z})/\sqrt{2}} - \rho_{00}^{\hat{x}}$$

- Spin alignment in $\mathbf{n}$ -direction

$$\begin{aligned} \rho_{00}^{\mathbf{n}} = & \frac{1}{8} \left\{ 3 - \rho_{00}^{\hat{y}} - 2\text{Re}\rho_{1,-1}^{\hat{y}} + (1 - 3\rho_{00}^{\hat{y}} - 6\text{Re}\rho_{1,-1}^{\hat{y}}) \cos 2\theta_{\mathbf{n}} \right. \\ & + 4 \sin 2\theta_{\mathbf{n}} \left(2\text{Im}\rho_{1,-1}^{\hat{y}} \cos \phi_{\mathbf{n}} - \sqrt{2}\text{Re}(\rho_{10}^{\hat{y}} - \rho_{0,-1}^{\hat{y}}) \sin \phi_{\mathbf{n}} \right) \\ & \left. + 2 \sin^2 \theta_{\mathbf{n}} \left[\left(1 + 2\text{Re}\rho_{1,-1}^{\hat{y}} - 3\rho_{00}^{\hat{y}} \right) \cos 2\phi_{\mathbf{n}} + 2\sqrt{2}\text{Im}(\rho_{10}^{\hat{y}} - \rho_{0,-1}^{\hat{y}}) \sin 2\phi_{\mathbf{n}} \right] \right\} \end{aligned}$$

- Tensor polarization can be expressed in terms of

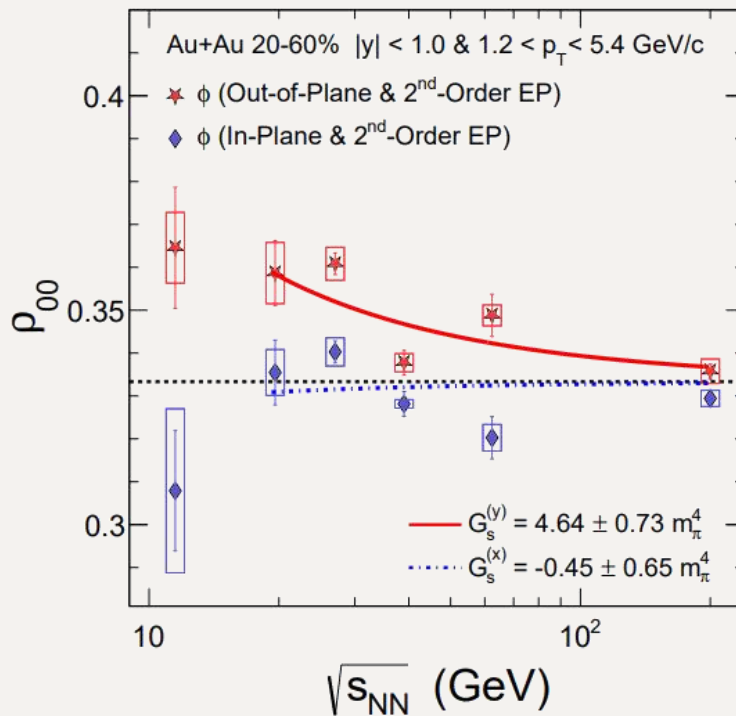
$$\begin{array}{ccccccc} \rho_{00}^{\hat{x}} & \rho_{00}^{\hat{y}} & \rho_{00}^{\hat{z}} & \rho_{00}^{(\hat{x}+\hat{y})/\sqrt{2}} & \rho_{00}^{(\hat{x}+\hat{z})/\sqrt{2}} & \rho_{00}^{(\hat{y}+\hat{z})/\sqrt{2}} \\ \underbrace{\hspace{1.5cm}} & & & & & \\ \rho_{00}^{\hat{x}} + \rho_{00}^{\hat{y}} + \rho_{00}^{\hat{z}} & = & 1, & & & \end{array}$$

Diagonal element

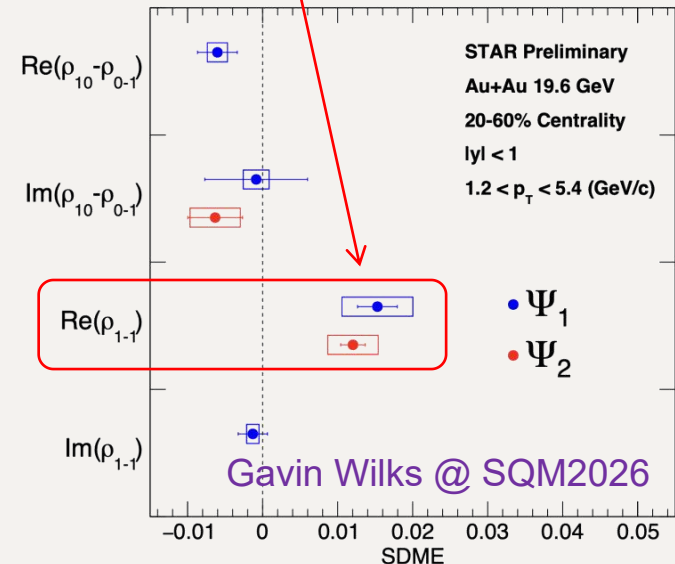
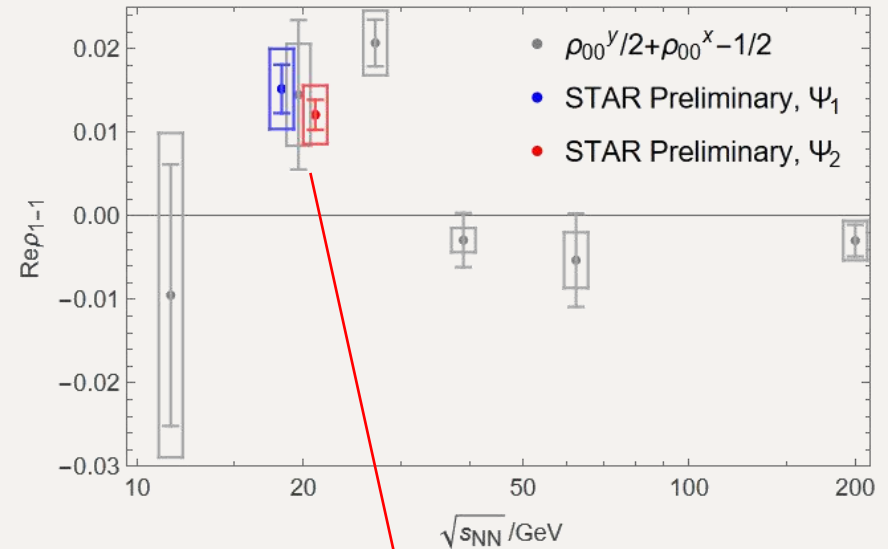


- Deriving off-diagonal element and spin alignment in beam direction using present data

$$\text{Re}\rho_{1,-1}^{\hat{y}} = \frac{1}{2}\rho_{00}^{\hat{y}} + \rho_{00}^{\hat{x}} - \frac{1}{2}$$



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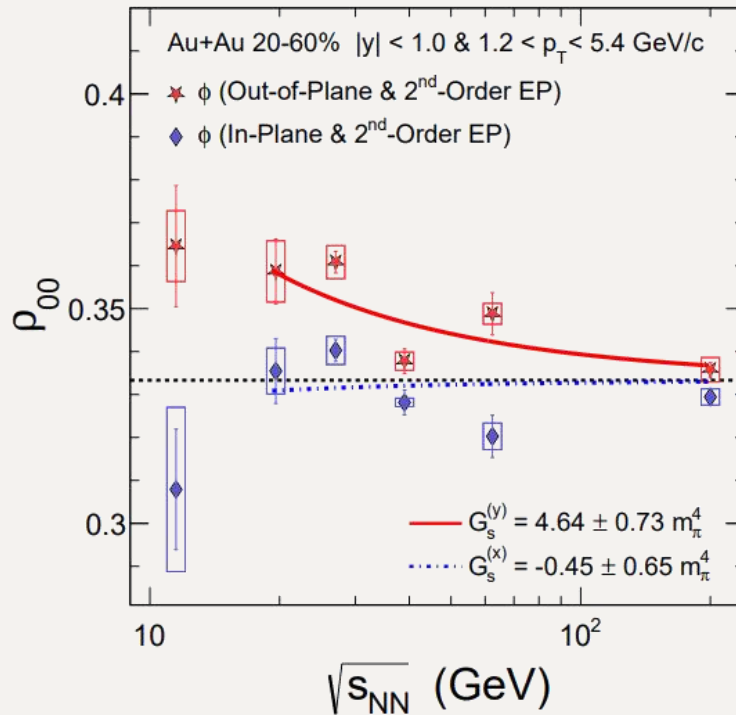


Spin alignment in beam direction

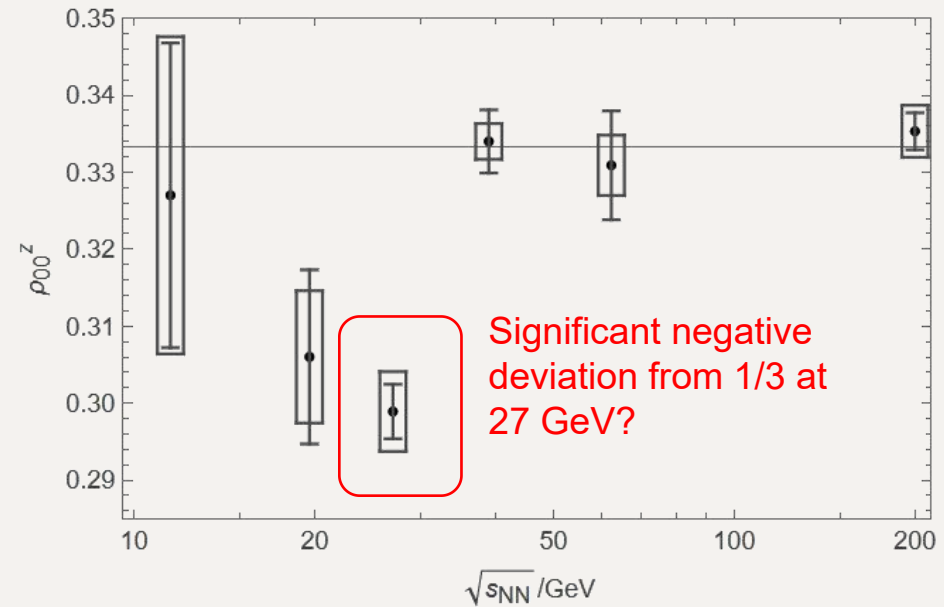


- Deriving off-diagonal element and spin alignment in beam direction using present data

$$\rho_{00}^{\hat{x}} + \rho_{00}^{\hat{y}} + \rho_{00}^{\hat{z}} = 1,$$



STAR, Nature 614, 244 (2023)



XLS, S. Lin, in preparation

General form of tensor polarization



- Spin density matrix

$$\rho^{\mathbf{n}} = \frac{1}{3}I_3 + \frac{1}{2}\mathbf{P}^{\mathbf{n}} \cdot \mathbf{S} + \frac{1}{4}\mathcal{T}_{ij}^{\mathbf{n}}\Sigma_{ij}$$

- Without loss of generality, we can study tensor polarization when quantization direction is along z-axis

$$\mathcal{T}_{ij}^{\hat{\mathbf{n}}} = R_{ik}R_{jl}\mathcal{T}_{kl}^{\hat{\mathbf{z}}} \quad R = \begin{pmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \mathbf{n} \end{pmatrix} \quad \mathbf{n}_1 \perp \mathbf{n}_2 \perp \mathbf{n}$$

$$\mathcal{T}_{kl}^{\hat{\mathbf{z}}} \equiv -2Q_{kl} \quad l = 2 \text{ irreducible tensor under SO}(3)$$

$$\rho_{00}^{\mathbf{n}} = \frac{1}{3} - \frac{1}{2}\mathcal{T}_{33}^{\mathbf{n}} = n_i n_j Q_{ij}$$

- Tensor polarization is **pairty-even**

Type	Examples	Pairty
Scalar	$T, \mu_B, \epsilon, P, \partial_\mu u^\mu, G_{\mu\nu}^a G^{a\mu\nu}$	even
Pseudoscalar	$\mu_5, \mathbf{V} \cdot \mathbf{A}$	odd
Polar vector	$\mathbf{u}, \nabla T, \nabla \mu, \mathbf{a}, \mathbf{E}, \mathbf{E}^a$	odd
Axial vector	$\varpi, \mathbf{B}, \mathbf{B}^a$	even
Pairty-even tensor	$\sigma_{(ij)}, V_{(i}^{(1)} V_{j)}^{(2)}, A_{(i}^{(1)} A_{j)}^{(2)}$	even
Pairty-odd tensor	$V_{(i} A_{j)}$	odd

* Fields in meson's rest frame

- General formula for tensor polarization

$$\begin{aligned}\mathcal{T}_{ij}^{\hat{\mathbf{z}}} = & \kappa_T T_{ij} + \kappa_{AT} (\epsilon_{ikl} A_k T_{lj} + \epsilon_{jkl} A_k T_{li}) + \kappa_{VT} (\epsilon_{ikl} V_k T_{lj}^{\text{odd}} + \epsilon_{jkl} V_k T_{li}^{\text{odd}}) \\ & + \kappa_{TT} \left[\frac{1}{2} (T_{ik}^{(1)} T_{kj}^{(2)} + T_{ik}^{(2)} T_{kj}^{(1)}) - \frac{1}{3} \delta_{ij} T_{kl}^{(1)} T_{kl}^{(2)} \right] \\ & + \tilde{\kappa}_T T_{ij}^{\text{odd}} + \tilde{\kappa}_{VT} (\epsilon_{ikl} V_k T_{lj} + \epsilon_{jkl} V_k T_{li}) \\ & + \tilde{\kappa}_{TT} \left[\frac{1}{2} (T_{ik} T_{kj}^{\text{odd}} + T_{ik}^{\text{odd}} T_{kj}) - \frac{1}{3} \delta_{ij} T_{kl} T_{kl}^{\text{odd}} \right]\end{aligned}$$

- T_{ij} includes bilinear combination of two polar-vector fields

$$\frac{1}{2} (V_i^{(1)} V_j^{(2)} + V_j^{(1)} V_i^{(2)}) - \frac{1}{3} \delta_{ij} \mathbf{V}^{(1)} \cdot \mathbf{V}^{(2)}$$

- T_{ij}^{odd} includes $\frac{1}{2} (V_i A_j + V_j A_i) - \frac{1}{3} \delta_{ij} \mathbf{V} \cdot \mathbf{A}$

- Scalar coefficients κ_T , κ_{VT} , κ_{AT} , and κ_{TT}

- Pseudoscalar coefficients $\tilde{\kappa}_T$, $\tilde{\kappa}_{VT}$, and $\tilde{\kappa}_{TT}$

- Verify whether a formula for spin alignment is correct
- Coefficient needs model calculation

- Vector meson spin alignment
 - ↔ anisotropy of short-range quark spin correlation
 - ↔ strong-field fluctuations (vector meson field or color field)
 - anisotropy of quark/antiquark distribution

- Off-diagonal elements
 - ↔ spin alignment in other directions

$$\begin{aligned} \text{Re}\rho_{1,-1}^{\hat{y}} &= \frac{1}{2}\rho_{00}^{\hat{y}} + \rho_{00}^{\hat{x}} - \frac{1}{2} & -\sqrt{2}\text{Im}(\rho_{10}^{\hat{y}} - \rho_{0,-1}^{\hat{y}}) &= 1 - 2\rho_{00}^{(\hat{x}+\hat{y})/\sqrt{2}} - \rho_{00}^{\hat{z}} \\ -2\text{Im}\rho_{1,-1}^{\hat{y}} &= 1 - 2\rho_{00}^{(\hat{x}+\hat{z})/\sqrt{2}} - \rho_{00}^{\hat{y}} & \sqrt{2}\text{Re}(\rho_{10}^{\hat{y}} - \rho_{0,-1}^{\hat{y}}) &= 1 - 2\rho_{00}^{(\hat{y}+\hat{z})/\sqrt{2}} - \rho_{00}^{\hat{x}} \end{aligned}$$

- General expression for vector meson tensor polarization

$$\begin{aligned} \mathcal{T}_{ij}^{\hat{z}} &= \kappa_T T_{ij} + \kappa_{AT} (\epsilon_{ikl} A_k T_{lj} + \epsilon_{jkl} A_k T_{li}) + \kappa_{VT} (\epsilon_{ikl} V_k T_{lj}^{\text{odd}} + \epsilon_{jkl} V_k T_{li}^{\text{odd}}) \\ &\quad + \kappa_{TT} \left[\frac{1}{2} (T_{ik}^{(1)} T_{kj}^{(2)} + T_{ik}^{(2)} T_{kj}^{(1)}) - \frac{1}{3} \delta_{ij} T_{kl}^{(1)} T_{kl}^{(2)} \right] \end{aligned}$$

Thanks for your attention!!

Choices of quantization axis



- Helicity (HX) $\frac{1}{|\mathbf{p}|} (p_x, p_y, p_z)$

Collins-Soper $\frac{E}{\sqrt{m^2 + p_T^2}} \left[(0, 0, 1) - \frac{p_z}{E(E + m)} (p_x, p_y, p_z) \right] \quad Y_{\text{beam}} \gg 1$

Gottfried-Jackson $\frac{m}{E - p_z} \left[(0, 0, 1) + \frac{p_z - E - m}{m(E + m)} (p_x, p_y, p_z) \right]$

Production plane

$$\frac{1}{p_T} (-p_y, p_x, 0)$$

Out-of-plane

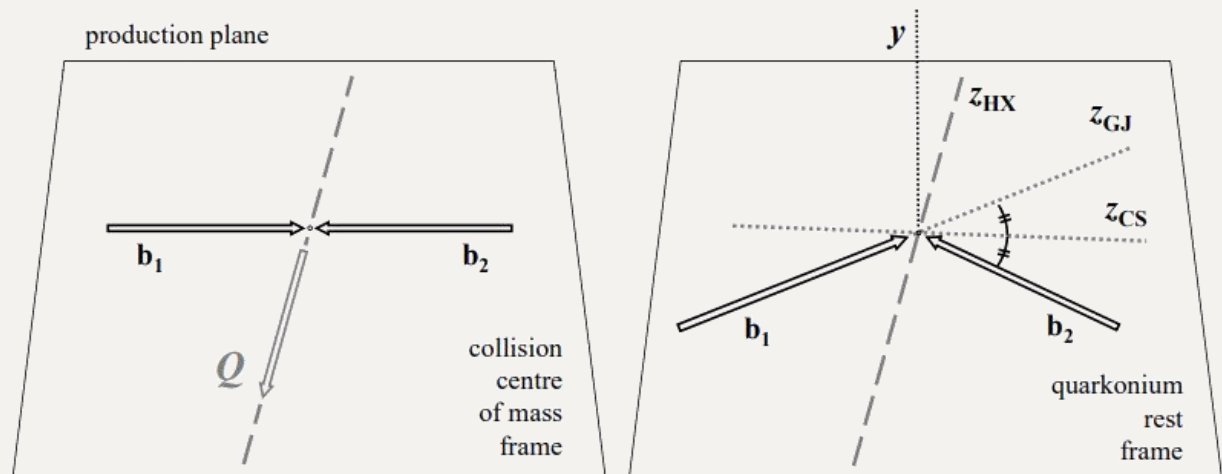
$$(0, 1, 0)$$

In-plane

$$(1, 0, 0)$$

Beam

$$(0, 0, 1)$$



P. Faccioli, C. Lourenco, J. Seixas, H. K. Wohri, EPJC 69, 657-673 (2010)